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Subject to Interest Rate Risk: An
Application of Raviv's Insurance Model

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Rate Risk: An Application of Raviv's Insurance Model

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Abstract

This paper derives the structure of the optimal variable-rate loan contract. The lender is assumed to rely on short-term borrowing in extending long-term loans, which means that the terms of the contract must be set before the uncertain future cost of funds is known. As a result, the contract specifies a "loan-rate" function $r(s)$, which gives the future loan interest rate r as a function of the future cost of funds s . The loan-rate function is chosen to maximize the lender's expected utility given a fixed level of expected utility for the borrower. The resulting optimal control problem is very similar to the one solved by Raviv (AER 1979) in analysing the optimal insurance contract. The central result of the analysis is that the slope $\dot{r}(s)$ of the loan-rate function satisfies $\dot{r} = \sigma_v / (\sigma_u + \sigma_v)$, where σ_u and σ_v are the absolute risk aversion measures for the borrower and lender respectively. This result shows that it is optimal for the lender (borrower) to bear more risk (for $\dot{r}$ to be lower (higher)) the more risk averse is the borrower (lender) (the higher is σ_u (σ_v)). This result is also generalized in a number of directions.

Efficient Contracts in Credit Markets Subject to Interest Rate Risk:
An Application of Raviv's Insurance Model

by

Lanny Arvan and Jan K. Brueckner*

1. Introduction

Interest rate risk borne by financial institutions has increased markedly as the credit markets have become more unstable in recent years. The burden of this new instability has been especially severe for lending institutions whose balance sheets show the greatest mismatch between the maturities of assets and liabilities. The savings and loan industry, for example, which borrows short and lends long, suffered serious losses in recent years as income from its portfolio of old mortgages failed to keep pace with the escalating cost of short-term funds. The specter of such losses has led to the emergence of adjustable-rate mortgages (ARMs) as well as other types of variable-rate contracts. These contracts often specify a complex functional relationship between the future loan interest rate and the lender's (uncertain) future cost of funds.

In view of the growing popularity of variable-rate loans, it is important to know what features an efficient contract of this type would possess. To answer this question, the present paper derives the form of the optimal variable-rate loan contract, focusing especially on the contractual relationship between the loan rate and the future cost of funds. The results of the analysis are especially relevant for today's mortgage market since they imply that the risk-sharing arrangement involved in the most popular ARMs (those with interest rate caps)

is inefficient. The analysis applies, however, to any credit market where lenders rely on short-term borrowing in extending loans of longer maturity.¹

The framework employed in the analysis is very similar to that used by Raviv (1979) to analyze the optimal insurance contract. In Raviv's model, the insured party incurs a loss of random size x , for which he receives an insurance payment according to the "coverage" function $I(x)$. In the model developed below, the lender (who plays the role of the insured party) suffers a "loss" s in the uncertain future period for each long-term dollar loaned. This loss represents the interest cost of the short-term funds used to extend long-term loans. The lender's loss is "covered" by the future interest payment on the loan, which is related to the uncertain cost of funds by the "loan-rate" function $r(s)$. Since the interest payment based on $r(s)$ represents a cost to the borrower, he plays the role of the insurer in the loan model. As will be seen below, parallel structure of the models means that Raviv's approach can be used to derive the optimal loan-rate function.

2. Analysis

In Raviv's model, the insurance policy requires a premium payment P in return for coverage of the random loss x according to the function $I(x)$. The insured party's net income conditional on x is thus $W + I(x) - x - P$, where W is exogenous income. An insurance settlement of size I is assumed to impose administrative costs of $c(I)$, resulting in a net income for the insurer equal to $w + P - I(x) - c(I(x))$ (w is exogenous income). The optimization problem is to choose P and the

function $I(\cdot)$ to maximize the insured party's expected utility while providing a given level of expected utility to the insurer. A constraint on the problem is the requirement $0 \leq I(x) \leq x$, which says that the insurance payment cannot be negative and must be no larger than the loss.

The loan model has a single borrower and a single lender who extends a two-period loan of exogenous size L . For simplicity, it is assumed that the borrower pays only interest on the loan at the end of each period, with the principal repaid at the end of the second period. The lender is assumed to rely on short-term borrowing for his loanable funds, which means the terms of the loan contract must be set before the cost of funds in the uncertain second period is known. As a result, the contract specifies a loan-rate function $r(s)$, which gives the second period loan interest rate as a function of the short-term rate that actually prevails in that period. The lender's net income in the second period is thus $(r(s) - s)L$, with net income in the first period equal to $(r_0 - s_0)L$ (s_0 is the first period's short-term rate, which is freely observable, while r_0 is the loan rate). Letting y_0 and y denote the borrower's incomes in the two periods, net incomes are $y_0 - r_0L$ and $y - r(s)L$ in the first and second periods respectively (y is non-random).² Inflation is assumed to be absent, so that all net incomes are in real terms.³

The parallel structure of the loan and insurance models is clear from the above discussion. First, recalling the analogy between s and x and between r and I , comparison of the relevant expressions shows that the lender's second-period income is similar in form to that of

the insured party (the equivalence is exact if $W = P = 0$ and $L = 1$). Similarly, the borrower's second-period net income is similar in form to that of the insurer (the equivalence is again exact if $P = 0$, $L = 1$, and $c(I) \equiv 0$). This suggests that Raviv's results on the optimal insurance coverage function will apply, at least in part, to the optimal loan-rate function. The following analysis shows that this is indeed the case.

Letting v denote the lender's utility function (which satisfies $v'' < 0$), δ denote his discount rate, and $f(s)$ denote the density function for s , the lender's discounted expected utility equals

$$v[(r_0 - s_0)L] + \delta \int_0^{\bar{s}} v[(r(s) - s)L]f(s)ds \quad (1)$$

($\bar{s}$ is the maximum value of s). The loan contract (which specifies r_0 as well the loan-rate function $r(\cdot)$) is chosen to maximize (1) subject to the requirement that the borrower's discounted expected utility equals some constant. Letting u denote the borrower's concave utility function and θ denote his discount rate, this constraint is written

$$u(y_0 - r_0 L) + \theta \int_0^{\bar{s}} u(y - r(s)L)f(s)ds = k, \quad (2)$$

where k is a constant. The maximization problem is solved by choosing the loan-rate function optimally conditional on r_0 and then optimizing over r_0 . Under this procedure, $r(\cdot)$ is chosen to maximize the expected utility integral in (1) subject to the requirement that the expected utility integral in (2) equals the constant $(k - u(y_0 - r_0 L))/\theta$.

Unlike in Raviv's insurance model, there is no upper bound constraint on $r(\cdot)$. Furthermore, while the constraint $r(s) \geq 0$ is

clearly appropriate, there is no reason to expect it to bind in the present model (the loan contract would offer free interest in this case). As a result, the nonnegativity constraint (which was crucial in yielding a deductible in Raviv's optimal insurance policy) can be ignored.

Following Raviv, the Hamiltonian for the problem of choosing $r(\cdot)$ is $\{v[(r(s) - s)L] + \lambda u(y - r(s)L)\}f(s)$, where λ is the costate variable. The optimality conditions are $\dot{\lambda} \equiv d\lambda/ds = 0$ and

$$v'[(r(s) - s)L] - \lambda u'(y - r(s)L) = 0, \quad (3)$$

which show that the optimal loan-rate function equates the lender's marginal utility to a constant proportion of the borrower's marginal utility regardless of the realized value of s . Using (3), it can be shown that optimal choice of r_0 yields a standard intertemporal efficiency condition.⁴ The slope of the loan-rate function is found by differentiating (3) with respect to s , which gives $(\dot{r} - 1)v'' + \lambda \dot{r}u'' = 0$. Eliminating λ using (3) and solving yields

$$\dot{r} = \frac{\sigma_v}{\sigma_u + \sigma_v}, \quad (4)$$

where $\sigma_v \equiv -v''/v'$ and $\sigma_u \equiv -u''/u'$ are the absolute risk aversion measures for the lender and borrower respectively (the σ 's are evaluated at the relevant net income levels). Raviv's eq. (10), which gives the slope of the insurance coverage function when $0 < I(x) < x$, reduces to (4) when $c' \equiv 0$.

Eq. (4) expresses a simple and intuitively appealing rule for the sharing of interest rate risk in credit markets. As one would expect, the equation shows that the slope of the optimal loan-rate function is positive and lies between zero and one, indicating that a percentage point change in the lender's cost of funds yields less than a percentage point change in the loan rate r . As a result, lender and borrower share the burden of a higher cost of funds as well as the benefit of a lower cost. More importantly, however, eq. (4) shows that the manner in which risk is allocated between lender and borrower depends on the relation between absolute risk aversion measures. If these measures are (locally) identical for the lender and borrower, then (4) shows that $\dot{r}(s)$ should equal $1/2$, indicating an equal (local) division of interest rate risk. If the lender is (locally) more risk averse than the borrower (if σ_v is greater than σ_u), then $\dot{r}(s)$ should exceed $1/2$, indicating that the borrower (locally) bears more of the interest rate risk. If the borrower is more risk averse than the lender (if $\sigma_u > \sigma_v$), then the reverse conclusion holds, with $\dot{r}(s) < 1/2$ and the lender bearing more of the risk. Polar cases emerge when one party is risk neutral. It is optimal for the borrower to bear all the interest rate risk if he is risk neutral ($\dot{r} = 1$ if $\sigma_u = 0$), while the lender should bear all the risk if he is risk neutral ($\dot{r} = 0$ if $\sigma_v = 0$). It is interesting to note that although the slope of the loan-rate function in general varies with s , it is constant when both lender and borrower exhibit constant absolute risk aversion. In this familiar case, the function is a straight line with slope between zero and one.

While the above analysis applies to any credit market where the maturities of the lender's assets and liabilities are mismatched, the analysis is especially useful in evaluating the growing tendency for risk-sharing in today's mortgage market. Prior to the 1980's, the fixed rate mortgage (which satisfies $\dot{r} = 0$) was the mainstay of the savings and loan industry. Although the lender inefficiently absorbs all risk under such a mortgage, the industry no doubt found such an arrangement acceptable given the relative stability of the market environment in which it operated. The heightened volatility of interest rates in recent years, however, inflicted large losses on the industry and gave rise to a greater desire for risk sharing. The result was the introduction of adjustable-rate mortgage (ARM), in which the borrower absorbs interest rate risk in return for a lower interest cost. This type of mortgage has become hugely popular in recent years, with ARMs accounting for over half of new mortgages written in 1984.⁵ The most common ARMs have interest rate caps, which prevent the loan rate from rising or falling excessively between adjustment periods.⁶ At each adjustment period, the loan rate is set equal to the short-term rate (or some cost of funds index) plus a markup, with the proviso that the resulting rate cannot differ from the previous period's loan rate by more than an amount given by the size of the cap. This relationship is shown by the jagged curve in Figure 1 (the horizontal segments are symmetric around the previous period's loan rate). While the ARM curve crudely approximates an optimal loan-rate function (shown as a dotted line in Figure 1), the requirement that interest rate risk be borne entirely by the lender for high and low values of s and entirely by the

borrower for intermediate values of s is incompatible with efficiency. Thus, although the ARM spreads risk more effectively than a fixed rate mortgage, the analysis implies that lenders could earn higher profits by offering ARM contracts satisfying the risk-sharing rule (4). While this conclusion is noteworthy, the simplicity of the model means that a negative efficiency verdict based on it should be viewed with some caution.

3. Extensions

Having established the usefulness of Raviv's approach in the credit market context, it is interesting to consider two extensions of the model. Under the first extension, the lender is allowed to choose his loan volume (the number of borrowers served). Under the second extension, the borrower's second period income y becomes random, with the analysis focusing on how the optimal risk-sharing arrangement changes when y is correlated with s . To explore the first extension, let n denote the number of borrowers served by the lender (the loan size L remains fixed for simplicity). Replacing L in (1) by nL , the first-order condition for choice of n is

$$(r_0 - s_0)v'_0 + \delta \int_0^{\bar{s}} (r(s) - s)v'f(s)ds = 0, \quad (5)$$

where v'_0 denotes the lender's marginal utility in the first period. Repeating the previous analysis, the slope of the loan rate function becomes

$$\dot{r} = \frac{n\sigma_v}{\sigma_u + n\sigma_v} \cdot \quad (6)$$

Although (6) is similar to the earlier slope formula (4), the appearance of the endogenous variable n on the RHS means that the connection between $\dot{r}$ and borrower and lender risk aversion is not as simple as before. While this complication makes the impact of a change in σ_u difficult to evaluate, it turns out that the effect of a change in σ_v is easily derived when absolute risk aversion is constant. In this case, it may be shown that $n\sigma_v$ (and hence $\dot{r}$) is invariant to a change in σ_v , with n falling as σ_v rises and vice versa.⁷ Since the intercept of the (linear) loan rate function and the value of r_0 are also invariant to a change in σ_v , it follows that the optimal loan contract is independent of the risk aversion of the lender. The only effect of an increase in risk aversion in the modified model is a reduction lender's optimal loan volume. This result is intuitively plausible given that the variability of the lender's net income $(r(s)-s)nL$ can be reduced in response to a higher σ_v either by an increase in $\dot{r}$ holding n fixed or by a reduction in n holding the loan-rate function fixed.

To investigate the effect of correlation between y and s , let $F(s,y)$ denote the joint density of these variables, with $G(y|s)$ denoting y 's conditional density. The borrower's expected utility in the second period becomes $\int_0^{\bar{s}} \int_0^{\bar{y}} u(y - r(s)L)F(s,y)dsdy$, and the Hamiltonian for the control problem (with $n = 1$) is rewritten as⁸

$$\{v[(r(s) - s)L] + \lambda \int_0^{\bar{y}} u(y - r(s)L)G(y|s)dy\}f(s). \quad (7)$$

With constant absolute risk aversion, the slope of the loan-rate function is given by

$$\dot{r} = \frac{\sigma_v - \Omega}{\sigma_u + \sigma_v}, \quad (8)$$

where

$$\Omega = \int_0^y u' \frac{\partial G}{\partial s} dy / L \int_0^y u' G dy. \quad (9)$$

If y and s are independent, so that $\partial G(y|s)/\partial s = 0$, then $\Omega = 0$ and (9) reduces to the previous expression (4). To investigate the dependence case, suppose that s and y are jointly normally distributed. In this case, $\partial G(y|s)/\partial s$ is proportional to $\rho(y - E(y|s))G(y|s)$, where ρ is the correlation coefficient between y and s ,⁹ and the numerator of Ω becomes proportional to $\rho \int_0^y u'(y - E(y|s))G(y|s)dy$. Since u' is a decreasing function of y and the remainder of the integrand integrates to zero, it follows that the integral multiplying ρ is negative. As a result, $\Omega > 0$ holds as $\rho < 0$. This in turn implies that, relative to the $\rho = 0$ case, a negative ρ lowers $\dot{r}$ while a positive ρ raises $\dot{r}$ (see (8)). Thus, if the borrower's income is positively (negatively) correlated with s , it is optimal for him to bear more (less) interest rate risk than when y and s are independent. The reason for these results is that, for a given loan-rate function, a positive (negative) ρ decreases (increases) the variability of the borrower's net income. To preserve an optimal balance between net income variabilities in the case where ρ rises above zero, the loan rate function must change to raise the variability of the borrower's interest costs while lowering the variability of the lender's net income (in other words, $\dot{r}$ must rise). The opposite change is called for when ρ falls below zero.

The above result is of practical interest since over the business cycle, interest rates and the incomes of certain types of borrowers are correlated. The incomes of workers in the residential construction industry, for example, appear to be negatively correlated with interest rates as a result of the inverse relationship between housing starts and borrowing costs. The analysis shows that optimal loan contracts should expose such workers to a relatively low degree of interest rate risk.

4. Conclusion

This paper has analyzed the optimal variable-rate loan contract using a framework similar to Raviv's optimal insurance model. In addition to showing that a variable-rate contract can be viewed as a type of insurance arrangement, the paper derived results of current practical interest. In particular, the optimal risk-sharing rule emerging from the model was shown to provide a useful starting point for evaluating the efficiency of existing variable-rate contracts. This is an important contribution since the variety of such contracts can be expected to multiply in today's increasingly innovative financial environment.

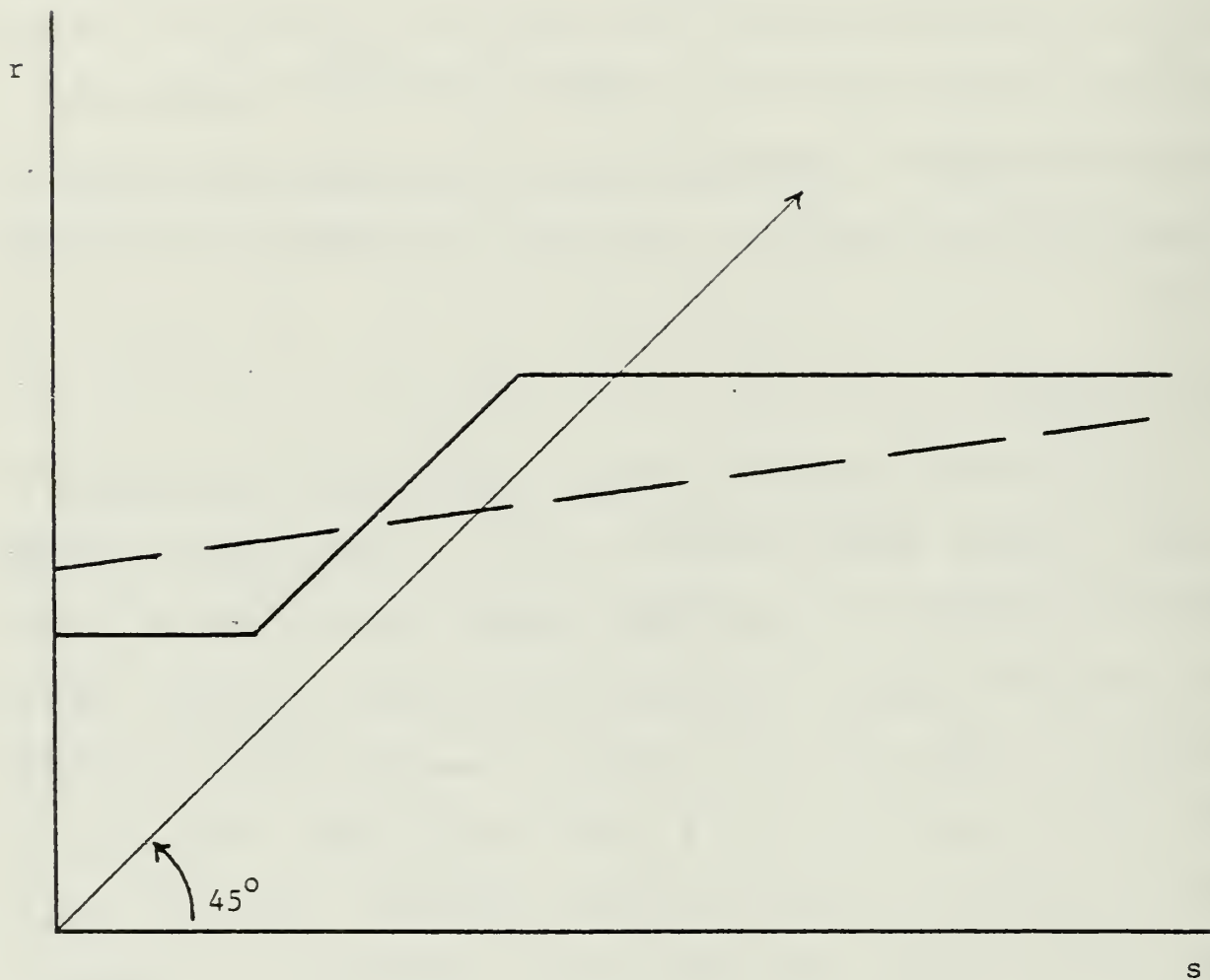


FIGURE 1.---A capped ARM vs. an optimal loan-rate function

Footnotes

*We wish to thank Case Sprenkle and James Follain for comments.

Errors are ours, however.

¹Several earlier papers deal with the maturity mismatch problem. Deshmukh et al. (1983a) and Niehans and Hewson (1976) analyse the lender's choice of the maturity structure of assets and liabilities in models without variable-rate loan contracts. Deshmukh et al. (1983a) analyze behavioral differences between lenders who both borrow and lend short and lenders who accumulate an initial stock of funds for lending to sequential loan applicants (such lenders in effect borrow long).

²Although the manner in which the borrower uses the loan is not relevant to the analysis, one possibility is that the funds are spent directly on consumption, in which case $y_0 = w_0 + L$ and $y = w - L$, where w_0 and w denote exogenous incomes. Alternatively, the proceeds of the loan could be used to purchase a capital good that generates incomes in the two periods according to the relationships $y_0 = h_0(L)$ and $y = h(L)$. Finally, the loan could finance purchase of a consumer durable such as a house. While utility in this case will depend on the services from the durable (as measured by L) as well as on net income, the service argument of the utility function is suppressed under the above formulation (note that in the latter two cases, the loan principal is repaid with the proceeds from the sale of the asset).

³The analysis is essentially unchanged when inflation is introduced provided that price increases are non-random. Stochastic inflation, however, changes the character of the results.

⁴The condition is

$$\frac{\delta \int_0^{\bar{s}} v' f ds}{v'_0} = \frac{\theta \int_0^{\bar{s}} u' f ds}{u'_0},$$

where v'_0 and u'_0 represent marginal utilities in the first period. This condition states that the expected marginal rates of substitution between net incomes in the two periods must be the same for lender and borrower.

⁵The Federal Home Loan Bank Board provides monthly market share data.

⁶Many ARMs also have life-of-loan interest rate caps, which limit cumulative rate adjustments.

⁷This can be seen by writing equations (2), (5), and the condition from footnote 4 for the constant absolute risk aversion (exponential utility) case using a linear r with slope (6). In the equations, n and σ_v only appear in the product expression $n\sigma_v$, establishing that if $\hat{n}$ solves the optimization problem with $\sigma_v = \hat{\sigma}_v$, then $\tilde{n} = \hat{n}\hat{\sigma}_v/\tilde{\sigma}_v$ solves the problem when $\sigma_v = \tilde{\sigma}_v$.

⁸If y were freely observable to the lender, then the optimal contract would make the loan rate contingent on both y and s , with a loan-rate function of the form $r(s,y)$. For the above formulation to represent a first-best optimum, y must be unobservable to the lender.

⁹Joint normality implies that $G(y|s)$ is proportional to

$$\exp\left\{-\frac{1}{2\varepsilon_y(1-\rho^2)} [y - \mu_y - \rho\varepsilon_y(s - \mu_s)/\varepsilon_s]^2\right\}$$

where μ_y and μ_s are the means of y and s and ε_y^2 and ε_s^2 are the variances.

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